

## Linear Transformations

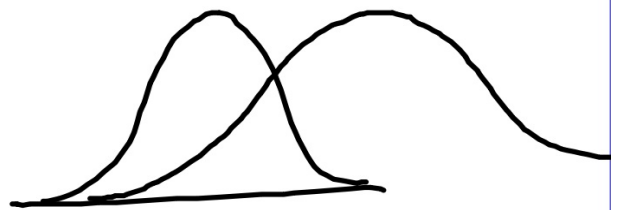
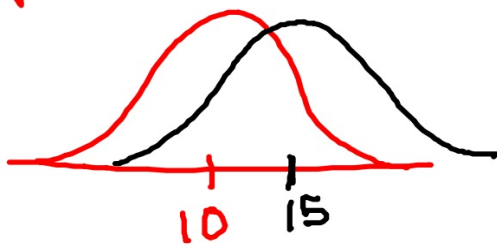
Shape = same

Center:  $+$ ,  $-$ ,  $\times$ ,  $\div$

Spread:  $\times$ ,  $\div$

$\times a$  all

$+b$  center only

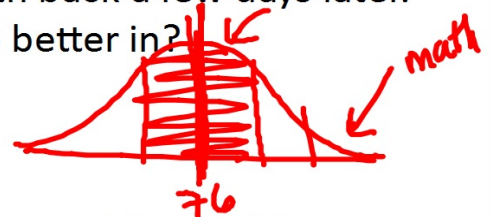


## Standardizing Observations

1) You are in a history class and a math class. You take your chapter 1 test in both classes on the same day, and get them both back a few days later. Your grades are as follows. Which class did you do better in?

History: 81%

Math: 75%



2) Same scenario. However, each teacher tells you a bit about how your class did. Which class did you do better in?

History: 81%

Math: 75%

mean: 76%

mean: 70%

either

3) Same scenario. However how your teacher gives you the class standard deviations. Which class did you do better in?

History: 81%

Math: 75%

mean: 76%

mean: 70%

std. dev: 8%

std. dev: 2.5%

less  
than  
15

25

## Standardizing Observations

**Question:** How can we compare one distribution to another distribution if they don't have the same parameters (mean and std. dev)?

**Answer:** compare the observation to its mean + std. dev.

**To standardize:**

- measure observations....

- $Z = \frac{X - \bar{X}}{S}$

- Z-score tells us...

how many S above or below  $\bar{X}$ .

in terms of how many std. dev. 's they are from mean.

$$\text{math} = \frac{75 - 70}{2.5} = 2$$

$$\text{hist} = 0.625$$

**Example:** The heights of 18-24 year old women are distributed with the following:

$\mu = 64.5''$  and  $\sigma = 2.5''$   $\mu$  = mean of a pop.

$\sigma$  = std. dev of pop.

We know a woman who is 69" tall. How does she compare to the rest of the women in her age group? Calculate her z score, and interpret what it means.

$$z = \frac{x - \mu}{\sigma} = \frac{69 - 64.5}{2.5} = 1.8 \sigma \text{ above } \mu$$

~~$\mu = 64.5$~~   
 ~~$\sigma = 2.5$~~

**Example:** We also know a man who is 71" tall. Who is taller relatively? Men are known to be distributed with a mean of 67" and a std. deviation of 2.3 inches.

$$\text{woman} = 1.8\sigma$$

$$\text{man} = 1.74\sigma$$

## Changing/Manipulating a set of data

What did we learn from the linear transformations wksht??

Adding/Subtracting a number just changes... *centers*

Dividing/Multiplying a number changes.... *center + spread*

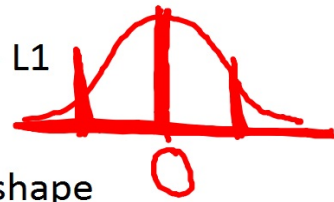
Did the shape change in any of these???

*NO!*

**Example:**

Set of data: {2, 3, 3, 4, 4, 4, 5, 5, 6} Type this into L1

$\bar{x}$   
0



- 1) Find the mean and std. deviation and the shape

$$\bar{x} = 4 \quad s = 1.2247 \quad \text{shape} = \text{Sym.}$$

- 2) Take the entire set of data and convert to z-scores (subtract the mean and then divide by the std. deviation). Do this in L2.

$$L2 = (L1 - 4) / 1.2247$$

- 3) Calculate the new mean and std. dev (of L2). Why is this so?

$$\bar{x} = 0 \quad s \approx 1$$

- 4) What does the shape of the distribution look like?

Symm.

This will happen with ANY distribution... When you change the whole thing into z-scores, your mean will be 0 and your std. dev. will be

1.



Graphs for Samples:

histograms, stemplots,  
etc.

Model for a population:



Described by STATISTICS

$\bar{X}, S, M$

Described by PARAMETERS

$\mu$   
 $\sigma$   
 ~~$\bar{x}$~~

Specific type of population... NORMAL



$N(64.5, 2.5)$

Notation:

$N(\underline{\mu}, \underline{\sigma})$

## Notes on Standardizing a distribution:

- Standardizing one observation...

compares it to its mean

- Standardizing a whole distribution allows us to....

- compare whole distrib. to its mean

⇒ + compare to another distrib.

- Normal distributions.... When we standardize them, we get the:

Standard Normal Distrib.

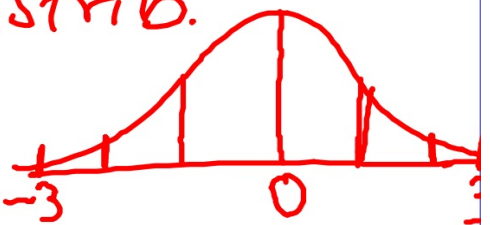
$$N(0, 1)$$

- Can only be used for...

nearly normal data/pop

- How can we check if data is NORMAL?

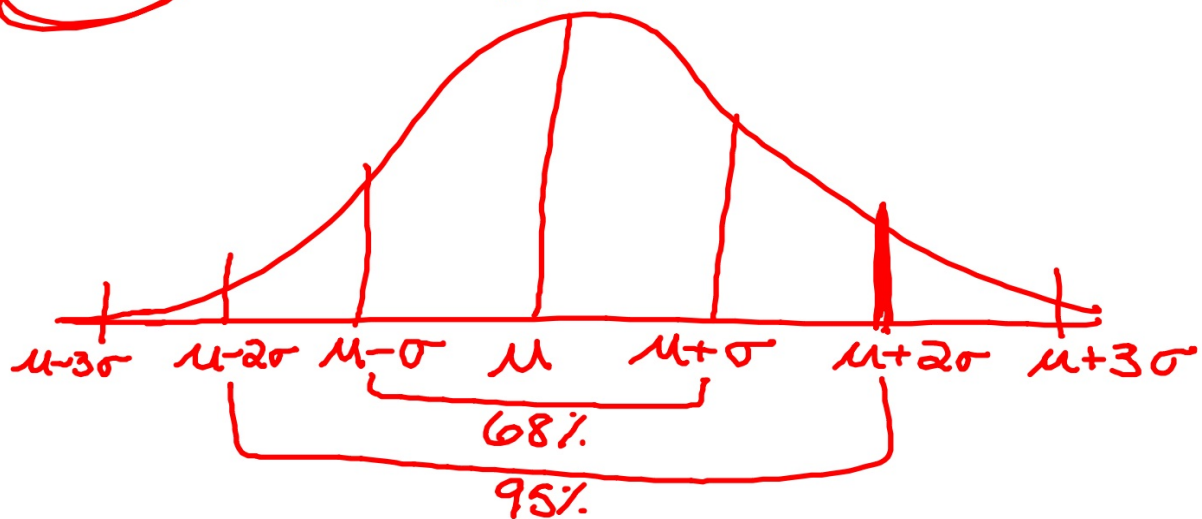
histogram





## Empirical Rule

In a normal distribution with  $N(\mu, \sigma)$ ....



- 68 % of the observations fall within  $\mu \pm \sigma$
- 95 % of the observations fall within  $\mu \pm 2\sigma$
- 99.7 % of the observations fall within  $\mu \pm 3\sigma$

**Example:**

The avg. time that it takes a crew from Sweeper Sam to clean a room is normally distributed with a mean of 2.1 hrs and a standard deviation of 0.3 hrs.

- a) The total clean up time will fall within what interval 95% of the time?
- b) What proportion of the time will it take the crew 2.5 hours or more?
- c) What percent of the time will it take the crew 1.5 hrs or less?

### **Back to the height example....**

Remember that the heights of 18-24 year old women are  $N(64.5", 2.5")$ . What percentile is the girl who is 68" tall?

### **Calculator:**

What percent of 18-24 year old women are less than 5 feet tall?

What percent 18-24 year old of women are over 5'8" tall?

## **\*\* PROBABILITY NOTATION!!**

### **Another example:**

Blood pressures of high school students are  $N(170, 30)$ . What is the probability that a randomly selected high school student has a blood pressure of 180 or above?

Using the same data as above, what is the probability that a high school student has a blood pressure between 160 and 230?

Using the same data as above, what blood pressure has 25% of the observations below it?

**Calculator use:**

**To find the PERCENT of observations BETWEEN 2 points:**

**To find what OBSERVATION has a certain percent of the data BELOW it:**

**On the calculator, INFINITY is:**

Complete worksheet 6A and check answers on the front desk!